Inequality and Growth in General Equilibrium

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Motivation

Workers might face frictions to move across jobs

- Due to costs, education, licenses, preferences, ...
- Workers do not move only following wages.

But our workhorse models don't allow for this

- Growth accounting: no labor mobility frictions.
- General equilibrium: either perfect mobility or perfect immobility.

This paper

- What is the impact of labor mobility frictions on aggregate growth and inequality?

What we do

Provide a simple model of growth accounting with mobility frictions

- Non-parametric, CRS production, homothetic demand.
- Aggregate output and welfare decline in mobility frictions.
- Characterized exactly by a change in the Theil index of wages.

Identify the impact of these frictions on growth and inequality in general equillibrium

- Multiple sectors, occupations, and input-output linkages.
- Shocks: shifts in labor productivity and mobility frictions.
- Channels: non-parametric first-order; parametric exact characterization.

Quantification on US data (1980-2019)

Growth accounting

- Change in inequality contributes up to 10-16% of decadal GDP growth.
- Wages decreases in previously 'important' sectors (with large income shares).

General equilibrium

- Simulate productivity/frictions shocks to all sectors on calibrated model.
- Workers' reallocation accounts for up to 40% of the change in real GDP.
- Δ inequality differs by up to 20 p.p. between low and high frictions settings.

Literature

Growth accounting: Solow 1956; Hulten 1978; Osotimehin 2019; Baqaee-Farhi 2020.

Misallocation: Kuznets 1961; Hsieh-Klenow 2009; Hopenhayn 2014; Peters 2020; Baqaee-Farhi 2020.

Macro and inequality: Keynes 1936; Bewley 1986; Huggett 1993; Aiyagari 1994; Kaplan et al. 2018; Auclert-Rognlie 2018; Bilbiie 2020.

Growth and inequality: Kuznets 1955; Kaldor 1957; Alesina-Rodrik 1994; Banerjee-Duflo 2003.

Wages in GE: Autor et al. 2003; Kambourov-Manovskii 2009; Acemoglu-Autor 2011; Acemoglu-Restrepo 2022; Jackson-Kanik 2019; Huneus et al. 2021: Lamadon et al., 2022.

Growth and human capital: Romer 1986; Lucas 1988; Deming 2022.

Today

Introduction

Growth accounting with mobility frictions

General equilibrium: non-parametric

General equilibrium: parametric examples

Quantification for the US

Setup

Aggregate output (real GDP) is given by

$$Y = zF(l_1, ..., l_{\mathcal{T}}, K) \text{ with } \sum_{t \in \mathcal{T}} l_t = L$$

- z: Hicks-neutral productivity.
- $F(\cdot)$: non-parametric CRS production function with diminishing returns.
- "Tasks" t are generic for now (e.g. occupation, industry,...).

Key: Exogenous frictions to how $l_1, ..., l_{\mathcal{T}}$ is allocated across labor types (e.g. mobility costs).

- Perfect mobility: $w_t = w$ for all $t \in \mathcal{T}$.
- Perfect immobility: w_t for all $t \in \mathcal{T}$.

Aggregate growth decomposition

First-order change in aggregate output $dlnY$ **around an initial equilibrium** $(P = 1)$

$$d\ln Y = d\ln z + \left(\frac{rK}{Y}\right) d\ln K + \sum_{t\in\mathcal{T}} \left(\frac{w_t l_t}{Y}\right) d\ln l_t$$

This is the classical growth accounting equation, with multiple labor types $t \in \mathcal{T}$.

Re-arranging and plugging in wages

$$d\ln Y = \underbrace{d\ln z}_{\Delta\ \text{Technology}} + \underbrace{\left(\frac{rK}{Y}\right) d\ln K}_{\Delta\ \text{Capital}} + \underbrace{\left(\frac{\tilde{w}L}{Y}\right) d\ln L}_{\Delta\ \text{Aggregate labor}} \underbrace{- \left(\frac{\tilde{w}L}{Y}\right) \sum_{t\in\mathcal{T}} \left(\frac{w_t l_t}{\tilde{w}L}\right) d\ln\left(\frac{w_t}{\tilde{w}}\right)}_{\text{Reallocation}}$$

where $\tilde{w} = \sum_{t\in\mathcal{T}} \frac{l_t}{L} w_t$ is the average wage.

Aggregate growth decomposition (cont'd)

$$d \ln Y = \underbrace{d \ln z}_{\Delta \text{ Technology}} + \underbrace{\Theta_K d \ln K}_{\Delta \text{ Capital}} + \underbrace{\Theta_L d \ln L}_{\Delta \text{ Aggregate labor}} \underbrace{- \Theta_L \sum_{t \in \mathcal{T}} \left(\frac{w_t l_t}{\tilde{w} L} \right) d \ln \left(\frac{w_t}{\tilde{w}} \right)}_{\text{Reallocation}}$$

where $\Theta_K = \frac{rK}{Y}$ and $\Theta_L = \frac{\tilde{w} L}{Y}$.

Intuition: a decrease in $(w_t / \tilde{w})$ between task-specific marginal productivity w_t and the average marginal productivity $\tilde{w}$ reflects a decline in the misallocation of workers.

Note:

- Aggregate labor supply effect $d \ln L$ and reallocation effect $d \ln \left(\frac{w_t}{\tilde{w}} \right)$.
- For $w_t / \tilde{w} = 0$, impact of reallocation $= 0$ (perfect mobility).
- For $w_t / \tilde{w} \gtrless 0$, impact of reallocation > 0.

Wage inequality as a Theil index

Theil index for income shares $\frac{w_t l_t}{\tilde{w} L}$

$$\mathcal{I}(\Theta) = \underbrace{\sum_{t\in\mathcal{T}} l_t \left(\frac{1}{L}\right) \ln\left(\frac{L}{1}\right)}_{\text{maximum entropy}} - \underbrace{\sum_{t\in\mathcal{T}} l_t \left(\frac{w_t}{\tilde{w}}\frac{1}{L}\right) \ln\left(\frac{\tilde{w}}{w_t}\frac{L}{1}\right)}_{\text{observed entropy}}$$

$$= \sum_{t\in\mathcal{T}} \left(\frac{w_t l_t}{\tilde{w} L}\right) \ln\left(\frac{w_t}{\tilde{w}}\right)$$

The Theil index is the entropy of wage ratios with labor income shares as weights.

The impact of mobility frictions is exactly characterized by the Theil index

$$d\ln Y = \underbrace{d\ln z}_{\Delta \text{ Technology}} + \underbrace{\Theta_K d\ln K}_{\Delta \text{ Capital}} + \underbrace{\Theta_L d\ln L}_{\Delta \text{ Aggregate labor}} \underbrace{-\Theta_L \sum_{t\in\mathcal{T}} \left(\frac{w_t l_t}{\tilde{w} L}\right) d\ln\left(\frac{w_t}{\tilde{w}}\right)}_{\Delta \text{ Theil index}}$$

Growth versus welfare

HH member h chooses which task t to work in based on wage and preferences

$$U^h = \max\left\{\varphi_1^h w_1, \cdots, \varphi_{\mathcal{T}}^h w_{\mathcal{T}}\right\}$$

where φ_t^h is a preference shifter for h supplying labor type t, from a Frechet distribution:

$$G \sim \exp\left\{\sum_{t\in\mathcal{T}} \phi_t \varphi_t^{-\kappa}\right\} \qquad \kappa > 1;\ \ \phi_t > 0,\ \ \forall t \in \mathcal{T}$$

ϕ_t (location): *absolute* attractiveness of each t. High $\phi_t \rightarrow$ type t is more attractive.
κ (shape): *relative* attractiveness. High $\kappa \rightarrow$ larger reallocation after shock.

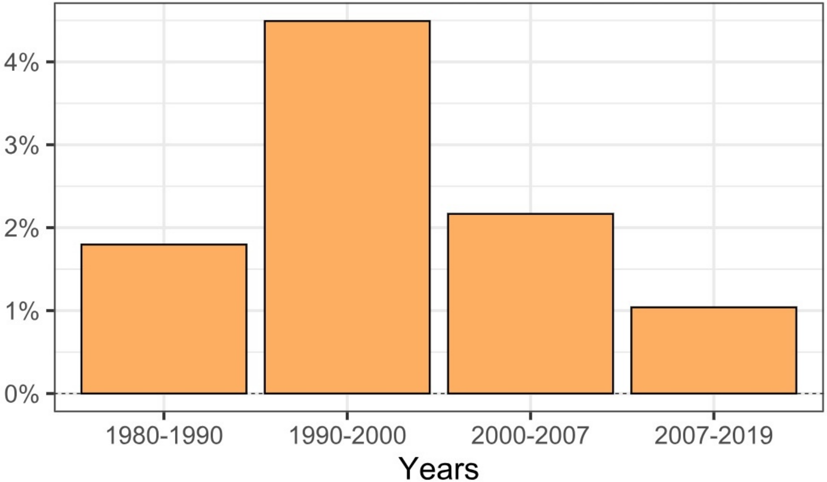
Welfare vs output growth:

$$d\ln U = d\ln\left(\sum_{t\in\mathcal{T}} \phi_t w_t^{1/\kappa}\right)^{\kappa} = d\ln Y + \sum_{t\in\mathcal{T}}\left(\frac{L_t}{L}\right) d\ln\Theta_t - d\ln L$$

Welfare and growth are both independent of workers' decisions related to preferences.

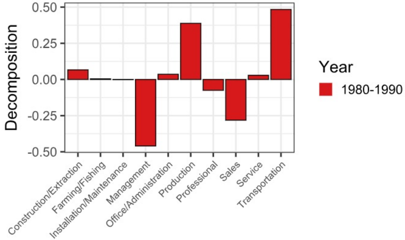
Change in US wage inequality (1980-2019)

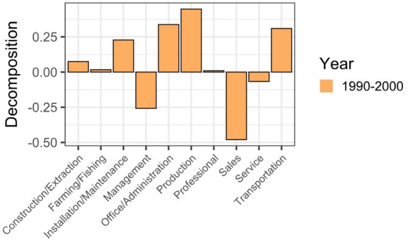
$$d\mathcal{I}(\Theta) = -\Theta_L \sum_{t\in\mathcal{T}} \left(\frac{w_t l_t}{\tilde{w} L}\right) d\ln\left(\frac{w_t}{\tilde{w}}\right)$$

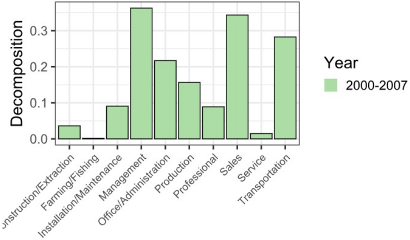


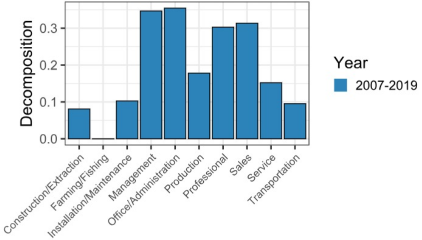
Contribution of occupations to inequality changes

Fraction of aggregate reallocation due to specific occupation t: $\dfrac{\left(\frac{w_t l_t}{\tilde{w} L}\right) d\ln(w_t/\tilde{w})}{\sum_{t\in\mathcal{T}}\left(\frac{w_t l_t}{\tilde{w} L}\right) d\ln(w_t/\tilde{w})}$ for each $t \in \mathcal{T}$









Today

Introduction

Growth accounting with mobility frictions

General equilibrium: non-parametric

General equilibrium: parametric examples

Quantification for the US

Overview

Context

- From the GA, we now can measure efficiency losses from labor mobility frictions.
- But what are the channels through which aggregate growth and inequality can occur?

General equilibrium framework

- Economy with multiple sectors $i \in \mathcal{N}$, occupations $o \in \mathcal{O}$, and factors L and K.
- Arbitrary elasticities of intermediates, factors, and occupations.

Production

- Sectors produce outputs using CRS technology on factors and inputs.
- Perfect competition.

Households

- Households consume with identical preferences.
- Choose labor and leisure subject to labor mobility frictions..

Production

Output for sector i, with constant returns to scale:

$$y_i = F_i(\underbrace{\{z_{io,L}\}_{o\in\mathcal{O}}}_{\text{input-specific shifters}}, \underbrace{\{l_{io}\}_{o\in\mathcal{O}}, k_i}_{\text{factor quantities}}, \underbrace{\{x_{ij}\}_{j\in\mathcal{N}}}_{\text{input quantities}})$$

Pricing for output i at marginal costs:

$$p_i = f_i\left(\{z_{io,L}\}_{o\in\mathcal{O}}, z_K, \{w_{io}\}_{o\in\mathcal{O}}, r, \{p_j\}_{j\in\mathcal{N}}\right)$$

Consumption

Total labor supply is determined by an aggregate utility function

$$\mathscr{U} = \underbrace{C}_{\text{consumption}} - \underbrace{h(L)}_{\text{disutility from working}}$$

Identical homothetic preferences:

$$Y = C\left(c_1, \cdots, c_{\mathcal{N}}\right)$$

budget constrained is pinned down by $w_{io} l_{io}$ for workers in occupation o and sector i.

Equilibrium

Equilibrium is defined by:

- capital$\{k_i\}_{i\in\mathcal{N}}$, and labor $\{l_{io}\}_{i\in\mathcal{N},o\in\mathcal{O}}$, intermediate goods $\{x_{ij}\}_{i,j\in\mathcal{N}}$,
- rent r, wages $\{w_{io}\}_{i\in\mathcal{N},o\in\mathcal{O}}$, and goods prices $\{p_i\}_{i\in\mathcal{N}}$,
- output $\{y_i\}_{i\in\mathcal{N}}$, and consumption levels $\{c_i\}_{i\in\mathcal{N}}$.

Determined jointly by the following conditions:

1. Firms maximize profits, equal to zero;
2. Workers maximize utility;
3. Labor, capital and goods markets clear:

$$L = \sum_o \sum_i l_{io}; \qquad K = \sum_i k_i; \qquad y_i = c_i + \sum_j x_{ji}$$

Given a vector of productivity shocks $\{z_{io,L}\}_{i\in\mathcal{N},\forall o\in\mathcal{O}}$ and capital supply K.

Shocks

Impact of labor productivity shocks $\{d \ln z_{io,L}\}$ on equilibrium outcomes

- Micro: prices $\{p_i,\ w_{io},\ r\}$ and quantities $\{l_{io}\ ,k_i,\ x_{ij}\}$ for all $i, j \in \mathcal{N}$ and $o \in \mathcal{O}$.
- Macro: real GDP Y, inequality I.

(Also results for shocks to mobility frictions in the paper)

Non-parametric vs parametric

- **Non-parametric:** up to first order.
- **Parametric:** full effects with nested CES.

Aggregate growth

The first-order impact of labor productivity shocks on growth is:

$$d \ln Y = \underbrace{\sum_i \sum_o \Theta_{io,L} d \ln z_{io,L}}_{\text{direct effect}} + \underbrace{\sum_i \sum_o \Theta_L \frac{d \ln L}{d \ln z_{io,L}} d \ln z_{io,L}}_{\text{aggregate labor}}$$

$$- \underbrace{\sum_i \sum_o \sum_j \sum_p \Theta_{jp,L} \frac{d \ln(w_{jp}/\tilde{w})}{d \ln z_{io,L}} d \ln z_{io,L}}_{\text{reallocation}}$$

Intuition

- **Direct effect**: a Harrod-neutral productivity shock affects real GDP proportional to its income share, $\Theta_{io,L}$ (Hulten, 1978).
- **Aggregate labor:** change in aggregate labor supply $d \ln L$, weighted by Θ_L.
- **Reallocation:** across sectors/occupations j, p from changes in i, o, weighted by $\Theta_{jp,L}$.

Wages

The effect of input-specific productivity shocks on wages is:

$$\frac{d \ln w_{jp}}{d \ln z_{io,L}} = \underbrace{\frac{d \ln \Theta_{jp,L}}{d \ln z_{io,L}}}_{\text{Labor demand}} - \underbrace{\frac{d \ln l_{jp}}{d \ln z_{io,L}}}_{\text{Labor supply}} + \underbrace{\frac{d \ln GDP}{d \ln z_{io,L}}}_{\text{Aggregate}}$$

Intuition:

- **Labor demand**: firms and consumers update choices, affect sector sizes and labor allocation through the networked economy.
- **Labor supply**: an increase in labor supply in a sector decreases wages in that sector.
- **Aggregate**: common shifter for all wages (only channel in Cobb Douglas economies!).

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Production and consumption

Production: firms produce following a sector-specific nested CES technology

$$y_i = \left(\omega_{i,L}^{\frac{1}{\sigma}} L_i^{\frac{\sigma-1}{\sigma}} + \omega_{i,K}^{\frac{1}{\sigma}} k_i^{\frac{\sigma-1}{\sigma}} + \sum_j \omega_{ij}^{\frac{1}{\sigma}} x_{ij}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \text{ with } L_i = \left(\sum_o \left(\frac{\omega_{io,L}}{\omega_{i,L}} \right)^{\frac{1}{\eta}} (z_{io,L} l_{io})^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$$

where $\sigma > 0$: elasticity of substitution between factors and intermediate goods and $\eta > 0$: elasticity between occupations.

Consumption: households have identical preferences across goods:

$$\mathscr{U}_c = \left(\sum_{i \in \mathcal{N}} \upsilon_i^{\frac{1}{\rho}} c_i^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}$$

where υ_i: preference shifter, and $\rho > 0$: elasticity of substitution across goods.

Labor preferences and frictions

Worker allocation: each household h draws a vector of preferences φ_{io}^{h}, drawn from a nested joint Fréchet distribution G:

$$G \sim \exp\left(\sum_{o\in\mathcal{O}} \nu_o \left(\sum_{i\in\mathcal{N}} \phi_{io}\left(\varphi_{io}\right)^{-\kappa}\right)^{\frac{\lambda}{\kappa}}\right)$$

with upper tier (ν, λ) and lower tier (ϕ, κ) location and dispersion parameters.

Share of workers specializing in occupation o in equilibrium is given by:

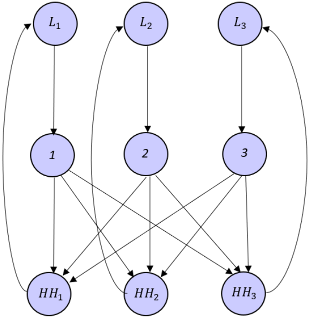
$$\Phi_o = \frac{l_o}{L} = \frac{\nu_o w_o^{\lambda}}{w^{\lambda}}$$

where $w = \left(\sum_{o\in\mathcal{O}} \nu_o w_o^{\lambda}\right)^{\frac{1}{\lambda}}$ is the wage index.

The **share of workers with occupation** o **supplying their labor to sector** i is equal to:$\Phi_{io} = \frac{l_{io}}{l_o} = \frac{\phi_{io} w_{io}^{\kappa}}{w_o^{\kappa}}$ where $l_o = \sum_i l_{io}$ and $w_o = \left(\sum_j \phi_{jo} w_{jo}^{\kappa}\right)^{\frac{1}{\kappa}}$.

Example 1: horizontal economy with fixed factors

Setup: Each HH supplies labor to one sector i, which uses only labor. Output sold to FD.



Results:

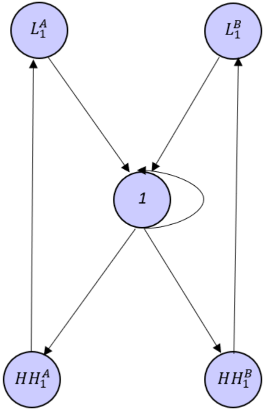
- Only HH h supplying labor in sector i is affected by changes in sales of i.
- Wage inequality depends only on ρ.
- Only a scale effect, no substitution effect:

$$d\ln\left(\frac{w_1}{w_2}\right) = \left(\frac{\rho - 1}{\rho}\right) d\ln\left(\frac{z_{1,L}}{z_{2,L}}\right)$$

- Only final demand elasticity ρ matters as production in each sector only uses labor.

Example 2: roundabout economy with fixed factors

Setup: Each HH supplies its labor type to only one sector. Output is sold to itself and to FD.



Results:

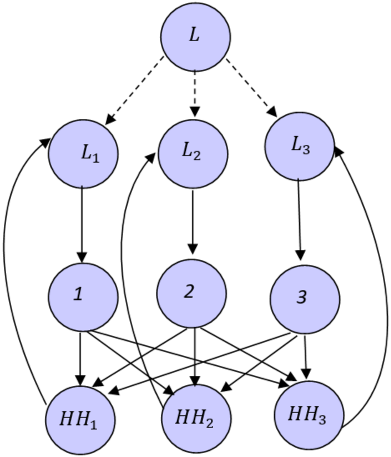
- Two labor types $\{A, B\}$ (e.g skilled/unskilled).
- Wage inequality depends only on σ.
- Only a substitution effect, no scale effect:

$$d \ln \left(\frac{w_{1A}}{w_{1B}} \right) = \left(\frac{\sigma - 1}{\sigma} \right) d \ln \left(\frac{z_{1A,L}}{z_{1B,L}} \right)$$

- There is only one final good, HH do not choose between different goods, their choices do not affect income inequality.

Example 3: horizontal economy with mobility frictions

Setup: Each HH supplies its labor type to only one sector. Output is sold to itself and to FD.



Results:

$$d \ln \left(\frac{w_1}{w_2} \right) = \left(\frac{\rho - 1}{\rho + \kappa} \right) d \ln \left(\frac{z_{1,L}}{z_{2,L}} \right)$$

If $\kappa = 0$, back to fixed factors.
If $\kappa \to \infty$, perfect mobility & no inequality.

General solution for any equilibrium network

$$\frac{d\ln w_{io}}{d\ln z_{sp,L}} = \underbrace{(\eta-\sigma)\frac{d\ln\left(\frac{w_{io}}{z_{io,L}}\right)}{d\ln z_{sp,L}} + (1-\eta)\frac{d\ln W_i^d}{d\ln z_{sp,L}} - (1-\sigma)\frac{d\ln p_i}{d\ln z_{sp,L}}}_{\text{Labor demand}}$$

$$\underbrace{+\frac{1}{\vartheta_i}(1-\rho)\sum_o \vartheta_{i,o}\frac{d\ln\left(\frac{p_o}{P}\right)}{d\ln z_{sp,L}} + \sum_k \frac{\vartheta_k}{\vartheta_i}(1-\sigma)\sum_o \theta_{ki,o}\frac{d\ln\left(\frac{p_o}{p_k}\right)}{d\ln z_{sp,L}}}_{\text{Labor demand}}$$

$$\underbrace{-\kappa\left(\frac{d\ln w_{io}}{d\ln z_{sp,L}} - \sum_j \left(\frac{l_{jo}}{l_o}\right)\frac{d\ln w_{jo}}{d\ln z_{sp,L}}\right) - \lambda\left(\frac{d\ln W_o^s}{d\ln z_{sp,L}} - \sum_j \left(\frac{l_j}{L}\right)\frac{d\ln W_j^s}{d\ln z_{sp,L}}\right)}_{\text{Labor supply}}$$

$$\underbrace{+\Theta_{sp,L} - \Theta_L \sum_j \sum_o \left(\frac{\Theta_{jo,L}}{\Theta_L}\right)\frac{d\ln\Gamma_{jo}}{d\ln z_{sp,L}}}_{\text{Aggregate channel}}$$

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Data and calibration

Data: IPUMS census data 1980-2019

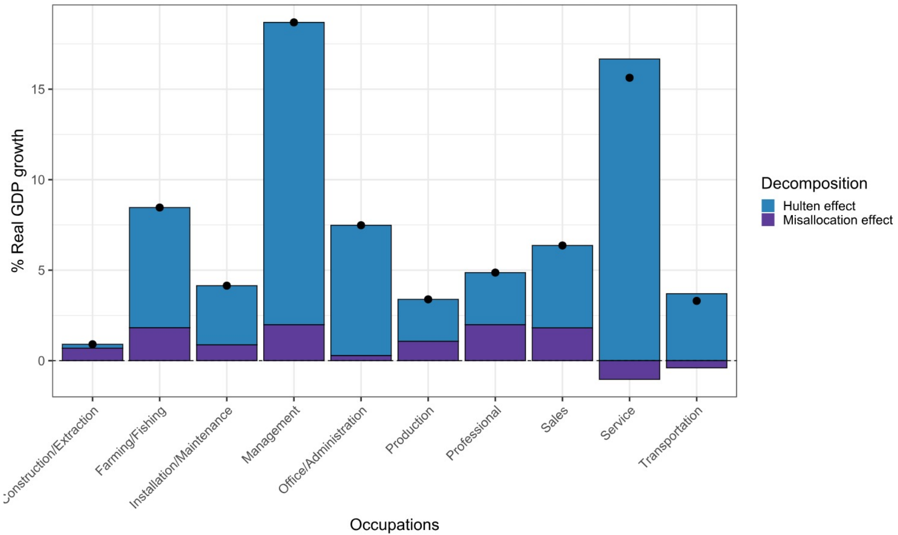
- 10 major occupations and 13 major sectors
- BEA input-output data for 2007

Calibration:

Description	Parameter	Value	Literature
Input elasticity	σ	0.5	[Atalay, 2017];
	η	0.5	[Oberfield and Raval, 2021]
Final demand elasticity	ρ	0.9	[Herrendorf et al., 2013]; [Oberfield and Raval, 2021]
Sectoral mobility elasticity	κ	1.4	[Galle and Lorentzen, 2023]

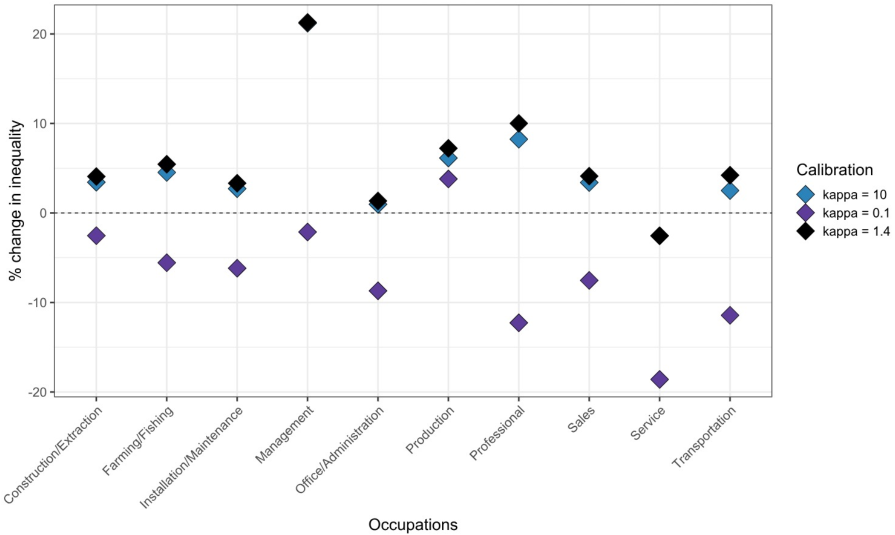
Imperfect workers' mobility matters for growth

The impact of a 10% productivity shock to an occupation on GDP



Imperfect workers' mobility matters for inequality

Differences in wage inequality can vary a lot across friction values



Conclusion

Labor mobility frictions hamper aggregate growth

- Due to misallocation of workers across occupations and sectors.
- Theil index as exact inequality measure from economic principles.

Frictions can affect growth and inequality through various channels

- Growth accounting with labor mobility frictions.
- GE framework of output growth and inequality.
- General non-parametric results up to first order and parametric exact characterizations.

Growth accounting results

- Change in inequality contributes up to 10-16% of decadal GDP growth.
- Wages decreases in previously 'important' sectors (with large income shares).

General equilibrium

- Workers' reallocation accounts for up to 40% of the change in real GDP.
- Δ inequality differs by up to 20 p.p. between low and high frictions settings.

Appendix

Appendix

- Appendix to intro: /
- Appendix to GA setup: change in Theil index
- Appendix to GA results: composition correction
- Appendix to GE setup: description variables and functions
- Appendix to wage description: 3 channels
- Appendix to GE calibration: some key calibration results (different economies, inequality-neutrality result,...)
- Appendix to GE results: additional results not shown in slides

Aggregate growth frictions as a change in the Theil index

A change in the Theil index is then (keeping income shares weights fixed)

$$
\begin{aligned}
dI\left(\bar{\Theta}\right) &= dQ(\mathcal{H}) - dQ(\bar{\Theta}) \\
&= \sum_{t\in\mathcal{T}} l_t \left(\frac{1}{L}\right) d\ln L - \sum_{t\in\mathcal{T}} \left(\frac{w_t l_t}{\tilde{w}L}\right) d\ln\left(\frac{L}{1}\frac{\tilde{w}}{w_t}\right) \\
&= \sum_{t\in\mathcal{T}} \left(\frac{w_t l_t}{\tilde{w}L}\right) d\ln\left(\frac{w_t}{\tilde{w}}\right)
\end{aligned}
$$

Key result 1: The previously *ad hoc* Theil index arrives naturally from simple growth theory!

Key result 2: $d\ln Y/dI(\bar{\Theta}) = -\Theta_L$ is the semi-elasticity of real GDP with respect to income inequality where $\Theta_L = \tilde{w}L/GDP$ is the total labor income share.

Result 1: Correction for composition effects

- We create demographic subgroups by education, age and gender
- We compute their respective labor share in each occupation in each period: $\omega_{ogt} = l_{ogt}/l_{ot}$
- We take the average labor share across periods: $\omega_{og} = \frac{1}{T}\sum_t \omega_{ogt}$
- We compute stable average wages:

$$w^*_{ot} = \sum_g \omega_{og} w_{ogt}; \qquad \tilde{w}^*_t = \sum_g \omega_g \tilde{w}_{gt}$$

Production

- **Dimensions:** $o \in \mathcal{O}$ labor types/occupations and $i \in \mathcal{N}$ sectors.
- **Output** for sector i, with constant returns to scale:

$$y_i = F_i\Big(\underbrace{\{z_{io,L}\}_{o\in\mathcal{O}}, z_K}_{\text{input-specific shifters}}, \underbrace{\{l_{io}\}_{o\in\mathcal{O}}, k_i}_{\text{factor quantities}}, \underbrace{\{x_{ij}\}_{j\in\mathcal{N}}}_{\text{input quantities}} \Big)$$

- **Pricing** for output i at marginal costs:

$$p_i = f_i\left(\{z_{io,L}\}_{o\in\mathcal{O}}, z_K, \{w_{io}\}_{o\in\mathcal{O}}, r, \{p_j\}_{j\in\mathcal{N}}\right)$$

Labor allocation

Workers are allocated across occupations o by maximizing a CRS allocation function:

$$L = G\left(l_1, \cdots, l_{\mathcal{O}}\right)$$

where l_o is the labor allocated to o.

Within each occupation, workers are allocated across sectors following $G_o(\cdot)$

$$l_o = G_o\left(l_{1o}, \cdots, l_{\mathcal{N}o}\right)$$

where l_{io} is the labor allocated to o in sector i.

Maximizing $G(\cdot)$ and $G_o(\cdot)$ wrt. the HH budget constraint, gives the **optimal allocation** across occupations and sectors as a function of wages and mobility costs:

$$\left(\frac{l_{1o}}{l_o}\right) = g_o\left(w_{1o}, \cdots, w_{\mathcal{N}o}\right); \qquad \left(\frac{l_o}{L}\right) = g\left(w_1, \cdots, w_{\mathcal{O}}\right)$$

which gives the following joint allocation per sector and occupation:

$$\left(\frac{l_{io}}{L}\right) = \left(\frac{l_{io}}{l_o}\right) \times \left(\frac{l_o}{L}\right) = g_o\left(w_{1o}, \cdots, w_{\mathcal{N}o}\right) \times g\left(w_1, \cdots, w_{\mathcal{O}}\right)$$

Utility and consumption

Total labor supply is determined by an aggregate utility function

$$\mathscr{U} = C - h(L)$$

where C is consumption and $h(L)$ reflects the disutility from working.

On the consumption side, households share identical homothetic preferences:

$$Y = C(c_1, \cdots, c_{\mathcal{N}})$$

where total income of the group of workers in occupation o and sector i is equal to $w_{io} l_{io}$, which pins down the budget constraint of each household.

Input-output definitions

Matrix elements $j \rightarrow i$

Tech coeffs: $\Omega_{ij,I} = \frac{p_j x_{ij}}{p_i y_i}$

Leontief inverse: $\Psi = (I - \Omega_I)^{-1}$

Labor share: $\Omega_{io,L} = \frac{w_{io} l_{io}}{p_i y_i}$

Capital share: $\Omega_{i,K} = \frac{r k_i}{p_i y_i}$

Final demand: $\Upsilon_i = \frac{p_i c_i}{GDP}$

Income share HH: $\Theta_{io,L} = \frac{w_{io} l_{io}}{GDP}$

		Use			Final demand
		1	$\cdots$	$\mathcal{N}$	
	1	$\Omega_{11,I}$	$\cdots$	$\Omega_{\mathcal{N}1,I}$	Υ_i
Supply	$\vdots$	$\vdots$	$\Omega_{ij,I}$	$\vdots$	$\vdots$
	$\mathcal{N}$	$\Omega_{1\mathcal{N},I}$	$\cdots$	$\Omega_{\mathcal{N}\mathcal{N},I}$	$\Upsilon_{\mathcal{N}}$
Labor	1	$\Omega_{11,L}$	$\cdots$	$\Omega_{\mathcal{N}1,L}$	
	$\vdots$	$\vdots$	$\Omega_{io,L}$	$\vdots$	
	$\mathcal{N} \times \mathcal{O}$	$\Omega_{1\mathcal{O},L}$	$\cdots$	$\Omega_{\mathcal{N}\mathcal{O},L}$	
Capital		$\Omega_{1,K}$	$\cdots$	$\Omega_{\mathcal{N},K}$	

The importance of sectors and HH in real GDP

Real GDP: $Y = C$

Supply centrality: The importance of sectors and HH for production is given by their *direct* and *indirect* importance in the production of final demand:

Sectors: through their Domar weights ϑ_i:

$$\vartheta_i \equiv \frac{p_i y_i}{GDP} = \sum_j \underbrace{\Upsilon_j}_{\textbf{FD share}} \underbrace{\Psi_{ji}}_{\textbf{Leontief inverse}}$$

$\rightarrow$ the importance of sector i as direct and indirect supplier to final demand.

Households: through their labor income shares $\Theta_{io,L}$:

$$\Theta_{io,L} \equiv \frac{w_{io} l_{io}}{GDP} = \sum_j \underbrace{\Upsilon_j}_{\text{FD share}} \underbrace{\Psi_{ji}}_{\text{Leontief inverse}} \underbrace{\Omega_{io,L}}_{\text{labor share}}$$

$\rightarrow$ the importance of labor in o in sector i as direct and indirect supplier to final demand.

The importance of sectors and HH in real GDP (cont'd)

Generalized Domar weights: Importance of a sector i to any other downstream sector m directly and indirectly through any j

$$\theta_{mi} = \sum_j \Omega_{mj,I} \Psi_{ji}$$

- i.e. θ_{mi} is a Domar weight, where the final demand is replaced by any sector m.

Demand centrality: Importance of a sector as a buyer to other sectors, and demand of labor directly and indirectly through any j

$$\xi_m = \sum_j \Psi_{mj} \qquad \text{(Leontief multiplier)}$$

$$\Xi_{mo,L} = \sum_j \Psi_{mj} \Omega_{jo,L} \qquad \text{(multiplier for labor demand } o \text{ in use of } m\text{)}$$

Prices

By Sheppard's lemma, we have that price changes are functions of the centrality measures:

$$d \ln p_i = \sum_j \sum_o \Xi_{ioL,j} d \ln \left(\frac{w_{jo}}{z_{jo,L}} \right) + \sum_j \Xi_{iK,j} d \ln r$$

Intuition: when a factor price w_{jo} changes, it impacts sector i's price p_i through i's direct and indirect importance as a *buyer* of that factor through $\Xi_{ioL,j}$.

Shock 2: Impact of productivity shocks on wage inequality

$$\frac{d\ln w_{io}}{d\ln z_{sp,L}} = \underbrace{\frac{d\ln \Theta_{io,L}}{d\ln z_{sp,L}}}_{\text{Labor demand channel}} - \underbrace{\frac{d\ln l_{io}}{d\ln z_{sp,L}}}_{\text{Labor supply channel}} + \underbrace{\frac{d\ln GDP}{d\ln z_{sp,L}}}_{\text{Aggregate channel}} \quad \forall i \in \mathcal{N}, \forall o \in \mathcal{O}$$

Labor demand channel

$$\frac{d\ln \Theta_{io,L}}{d\ln z_{sp,L}} = \underbrace{\frac{d\ln \Omega_{io,L}}{d\ln z_{sp,L}}}_{\textbf{labor use effect}} + \underbrace{\frac{d\ln \vartheta_i}{d\ln z_{sp,L}}}_{\textbf{scale effect}}$$

- Change in income shares $\Theta_{io,L}$ from a change in **labor use** $\Omega_{io,L} = \frac{w_{io} l_{io}}{p_i y_i}$ and/or a change in market **size** $d\ln \vartheta_i$.
- **Intuition**: New equilibrium prices and wages. Firms and consumers update choices and new labor shares and sector sizes result in equilibrium.

Shock 2: Impact of productivity shocks on wage inequality

$$\frac{d\ln w_{io}}{d\ln z_{sp,L}} = \underbrace{\frac{d\ln \Theta_{io,L}}{d\ln z_{sp,L}}}_{\text{Labor demand channel}} - \underbrace{\frac{d\ln l_{io}}{d\ln z_{sp,L}}}_{\text{Labor supply channel}} + \underbrace{\frac{d\ln GDP}{d\ln z_{sp,L}}}_{\text{Aggregate channel}} \quad \forall i \in \mathcal{N}, \forall o \in \mathcal{O}$$

Labor supply channel

$$\frac{d\ln l_{io}}{d\ln z_{sp,L}} = \underbrace{\frac{d\ln(l_{io}/l_i)}{d\ln z_{sp,L}}}_{\text{sectoral share}} + \underbrace{\frac{d\ln(l_o/L)}{d\ln z_{sp,L}}}_{\text{occupational share}} + \underbrace{\frac{d\ln L}{d\ln z_{sp,L}}}_{\text{Total labor supply}}$$

- Productivity shocks induce workers to move across sectors (with fixed ν and ϕ).
- Perfect immobility: no reallocation of workers, inducing large effects on income inequality.
- Perfect mobility: large reallocation, no wage inequality.

Shock 2: Impact of productivity shocks on wage inequality

Effect of input-specific productivity shocks $z_{sp,L}$, on wages w_{io}:

$$\frac{d \ln w_{io}}{d \ln z_{sp,L}} = \underbrace{\frac{d \ln \Theta_{io,L}}{d \ln z_{sp,L}}}_{\text{Labor demand channel}} - \underbrace{\frac{d \ln l_{io}}{d \ln z_{sp,L}}}_{\text{Labor supply channel}} + \underbrace{\frac{d \ln GDP}{d \ln z_{sp,L}}}_{\text{Aggregate channel}} \quad \forall i \in \mathcal{N}, \forall o \in \mathcal{O}$$

Aggregate channel

$$\frac{d \ln GDP}{d \ln z_{sp,L}} = \underbrace{\Theta_{sp,L}}_{\text{Productivity}} + \underbrace{\Theta_L \frac{d \ln L}{d \ln z_{sp,L}}}_{\text{Total labor supply}} \underbrace{- \Theta_L \sum_j \sum_o \left(\frac{\Theta_{jo,L}}{\Theta_L} \right) \frac{d \ln \Gamma_{jo}}{d \ln z_{sp,L}}}_{\text{Inequality}}$$

- Impact of productivity changes on aggregate output.
- Output shifter: no impact on inequality.
- This is the only channel in Cobb-Douglas economies.

Parametric form

Worker allocation: each household h draws a vector of idiosyncratic frictions φ_{io}^{h}, drawn from a nested joint Fréchet distribution G:

$$G \sim \exp\left(\sum_{o\in\mathcal{O}} \nu_o \left(\sum_{i\in\mathcal{N}} \phi_{io}\left(\varphi_{io}\right)^{-\kappa}\right)^{\frac{\lambda}{\kappa}}\right)$$

with upper tier Frechet (ν, λ) and lower tier (ϕ, κ) location and dispersion parameters.

In equilibrium, the **share of workers specializing in occupation** o is given by:

$$\Phi_o = \frac{l_o}{L} = \frac{\nu_o w_o^{\lambda}}{w^{\lambda}}$$

where $w = \left(\sum_o \nu_o w_o^{\lambda}\right)^{\frac{1}{\lambda}}$ is the wage index of workers.
The **share of workers with occupation** o **supplying their labor to sector** i is equal to:$\Phi_{io} = \frac{l_{io}}{l_o} = \frac{\phi_{io} w_{io}^{\kappa}}{w_o^{\kappa}}$ where $l_o = \sum_i l_{io}$ and $w_o = \left(\sum_j \phi_{jo} w_{jo}^{\kappa}\right)^{\frac{1}{\kappa}}$.

Data construction

Description	Parameter	Data construction
Technical coefficient matrix	Ω_I	BEA 13×13 matrix divided by sectoral gross output
Leontief matrix	Ψ	$(I - \Omega_I)^{-1}$
Labor share of cost	Ω_L	IPUMS average hourly wage multiplied by the number of hours worked in the sector
Capital share of cost	Ω_K	Value-added per sector from BEA data minus the labor share computed above

Data construction

Description	Parameter	Data construction
Final demand share	Υ	Simple share of total final demand allocated to a specific sector in BEA data
Domar weight	ϑ	Gross output divided by GDP computed as sum of value-added in BEA data
Labor income shares	Θ_L	$\Omega_L^{'}\vartheta$
Capital income share	Θ_K	$\Omega_K^{'}\vartheta$

Full characterization of the productivity shocks

- Prices and wage indices are formulated as functions of wages
- We obtain an equation of the form:

$$\mathrm{A} \, \mathrm{d} \ln \mathrm{w} = \mathrm{d} \ln \mathrm{z}$$

- where A is a 131×131 matrix, $\mathrm{d} \ln \mathrm{w}$ is a 131-element vector of wages and capital cost and where $\mathrm{d} \ln \mathrm{z}$ is a 131-elements vector of productivity shocks
- where each cell of the matrix A is given by:

$$
\begin{aligned}
\frac{d\ln w_{io}}{d\ln z_{sp,L}} = & \underbrace{(\eta-\sigma)\frac{d\ln\left(\frac{w_{io}}{z_{io,L}}\right)}{d\ln z_{sp,L}} + (1-\eta)\frac{d\ln W_i^d}{d\ln z_{sp,L}} - (1-\sigma)\frac{d\ln p_i}{d\ln z_{sp,L}}}_{\text{Labor demand}} \\
& \underbrace{+\frac{1}{\vartheta_i}(1-\rho)\sum_o \vartheta_{i,o}\frac{d\ln\left(\frac{p_o}{P}\right)}{d\ln z_{sp,L}} + \sum_k \frac{\vartheta_k}{\vartheta_i}(1-\sigma)\sum_o \theta_{ki,o}\frac{d\ln\left(\frac{p_o}{p_k}\right)}{d\ln z_{sp,L}}}_{\text{Labor demand}} \\
& \underbrace{-\kappa\left(\frac{d\ln w_{io}}{d\ln z_{sp,L}} - \sum_j\left(\frac{l_{jo}}{l_o}\right)\frac{d\ln w_{jo}}{d\ln z_{sp,L}}\right) - \lambda\left(\frac{d\ln W_o^s}{d\ln z_{sp,L}} - \sum_j\left(\frac{l_j}{L}\right)\frac{d\ln W_j^s}{d\ln z_{sp,L}}\right)}_{\text{Labor supply}} \\
& \underbrace{+\Theta_{sp,L} - \Theta_L\sum_j\sum_o\left(\frac{\Theta_{jo,L}}{\Theta_L}\right)\frac{d\ln \Gamma_{jo}}{d\ln z_{sp,L}}}_{\text{Aggregate channel}}
\end{aligned}
$$

Change in hourly wages