A new public goods' game: how heterogeneous agents invest in the green transition

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Abstract

We develop a new model for green investments which blends features of the portfolio choice model and the non-cooperative private provision of pubic goods game. Agents differ in time preference and in economic size. Our model shows that (a) long-term investors tend to have a larger share of their portfolio in green bonds, (b) larger agents/economies tend to invest more in climate mitigation, and (c) more heterogeneity among the types of agent (both in terms of time preference as well as economic size, looking beyond thresholds) increases aggregate climate investments. We provide empirical evidence for the first two effects. Our model has significant implications for climate policy: it points to the importance of long-term investors in the green transition and to the importance of larger groups of countries agreeing on cooperative frameworks, for example, in the form of climate clubs, to increase overall investments in climate mitigation.

1 Introduction

An extensive literature aims to understand why countries or agents invest (or do not invest) in climate-protection measures. Estimates find high economic consequences of climate change with correspondingly potentially large returns to climate mitigation measures (Bilal and Känzig, 2026). Yet, actual investments in climate mitigation remain low in most parts of the world. Climate measures such as carbon pricing rarely reach levels that would reflect the estimated social cost of carbon under even the more conservative estimates (Nath et al., 2024). Why then do countries and individuals invest too little? The answer is often framed as a global tragedy of the commons, in which missing property rights for the remaining carbon budget, that is, the amount of greenhouse gasses that can still be emitted while respecting certain goals for global temperature increases, result in individuals and countries emitting too much. Thus, the *commons* will be rapidly reduced and global climate change will accelerate.

While there is some debate about whether this conceptual framework captures well the reasons why it is so difficult to ensure global climate mitigation efforts are commensurate with the challenge

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(Aklin and Mildenberger, 2020), the tragedy-of-the-commons perspective has a long and influential tradition in political economy and other fields of economics. Originating with Hardin's (1968) seminal formulation, the framework emphasizes how individually rational behavior leads to collectively inefficient outcomes when a shared resource is unregulated or does not have a clearly established owner. Applied to climate change, greenhouse gas emissions can be thought of as arising from the absence of enforceable property rights over the remaining carbon budget, i.e., the amount of emissions that would be compatible with staying within certain global temperature limits. A large body of work formalizes this logic using public-goods and common-pool-resource models, highlighting incentives to free-ride on others' mitigation efforts (Barrett, 1994; Nordhaus, 2015; Weitzman, 2014). This framing has been widely adopted in both theoretical and applied analyses of international climate policy, including integrated assessment models and studies of climate agreements, see for example MacKay et al. (2015) and Tagliapietra and Wolff (2021).

We develop a model in that conceptual spirit. In particular, our framework abstracts from preference-based motives for environmental action, such as intrinsic "green" or moral preferences emphasized in the literature on warm-glow giving (Andreoni, 1990), and instead focuses on strategic behavior under standard preferences. Likewise, we do not rely on uncertainty about climate damages, abatement technologies, or future policy uncertainty (Hambel and van der Ploeg, 2025) as a primary driver of green investment, as is common in models that highlight risk, learning, or option value as central determinants of environmental investment decisions (Pindyck, 2007; Heal, 2009). Having said that, our approach does not contradict these two approaches as warm-glow motives and financial motives may play a role in addition to our private provision of public good game model.

Our model builds on a classic voluntary-contribution public goods game in the spirit of Bergstrom et al. (1986), in which agents contribute to a public good that benefits all players but is privately costly. We introduce one key modification by embedding this framework in a two-period setting. This extension allows us to incorporate intertemporal choice, savings, and investment decisions in a parsimonious way. By introducing time preference and capital accumulation, the model explicitly links financial decisions to green investments, enabling us to study how different shares of agents with different discounting factors affect the share of resources allocated to green investment. In doing so, the model highlights a novel channel through which heterogeneity in time preferences can shape equilibrium contributions to a global public good such as climate protection.

Our model makes three major predictions. First, more long-term oriented investors should have a greater propensity to invest in green assets. Second, larger actors/countries should invest more in green than their smaller counterparts. Finally, the model also predicts that as difference in economic size increases beyond a certain threshold, the share of total (and average) green investments should grow. Thus, the key predictions of our model reveal new interesting effects releva sustainable finance.

We then provide a simple econometric regression analysis that shows a positive correlation between long-term investment shares and green investment shares in financial and in non-financial sectors, using the ECB's securities holdings statistic. We also show that larger economies tend to increase green investments more than smaller countries in a set of 149 countries during 2020-23 using a dataset on renewable energy investments.

Our contribution to the literature is threefold. First, we add to the theoretical literature by developing a model of green investment decisions based on a marriage of the portfolio choice model and the private provision of public goods game with heterogeneous agents. Second, our model reveals the interactions between heterogeneity of agents across time preference and endowment with the portfolio choice of investing in a green or in conventional assets. We thus add to the theoretical literature on sustainable finance with a model based on a global public goods game. Finally, we provide new empirical evidence that supports the model's central findings of long-term time preference and economic size driving green investments.

The rest of the paper unfolds as follows. In Section 2 we develop a simple model of green investment and obtain our main results. Preferences in our baseline model are logarithmic, while subsection 2.5 extends the main results to the case of constant relative risk aversion (CRRA) preferences as a robustness check. Section 3 presents stylized facts that are consistent with the model's predictions based on a simple econometric model. Section 4 concludes. Appendix A provides technical proofs and details of the model, while Appendix B contains additional figures.

2 Model

In this section, we develop a simple model that highlights the role of investors' heterogeneity in time preference and size for the green transition.

The main feature of our model is that we treat green investment as a public good. The reason why green investment has features of a public good is because each agent's additional euro of green investment contributes to the prevention of an extreme climate event from which all agents would benefit. Through green investment, greenhouse gas emissions are reduced and thereby the global commons are protected. While the economic costs of extreme weather events is likely to further increase in the coming years (Bilal and Känzig, 2026; Carleton et al., 2022), the extent of the increase in severity crucially depends on how much greenhouse gas emissions will be reduced (Claeys et al., 2024). In our model, we simplify this complex relation to a world with two states: one in which the damage of climate change is relatively limited and one in which extreme events happen due to a further deterioration of climate change. For simplicity, we assume that the global climate deterioration affects the well-being of all agents in relation to their individual size. The green investment made by each agent therefore also has an effect on the agent itself. The green investment can thus be thought of as a privately supplied public good, in the spirit of (Bergstrom et al., 1986).

2.1 Model structure

Consider a world with n countries. Each country is modeled as its representative agent labeled as $i \in \{1, 2, \ldots, n\}$. Agents differ in two dimensions:

- time preference captured by the agent-specific discount factor $\beta_i \in (0, 1)$ (see Assumption 6 below for details);
- initial wealth w_i, of which in this endowment economy one can think as all the resources at the agent's disposal that can be spent.

There are two time periods: the present ($t = 0$) and the future ($t = 1$). Let c_{it} be consumption of agent $i \in \{1, 2, \ldots, n\}$ at period $t \in \{0, 1\}$, and let s_i be agent i's savings. The allocation

(c_{i0}, s_i) of the initial wealth between immediate consumption (which one can think of as short-term investment) and savings (long-term investment) must satisfy the standard budget constraint:

$$c_{i0} + s_i = w_i. \tag{1}$$

There are two types of assets in which agents can invest their savings: conventional bonds with returns R_b and green bonds with return R_g. Let b_i and g_i be, respectively, agent i's conventional and green holdings. Thus, agent i's asset portfolio (b_i, g_i) must satisfy

$$b_i + g_i = s_i, \tag{2}$$

while agent i's future consumption is given by

$$c_{i1} = R_b b_i + R_g g_i. \tag{3}$$

Agent i's protfolio structure is fully defined by the vector $(\gamma_i, 1-\gamma_i)$, where $\gamma_i \in [0, 1]$ is investor i's share of green holdings:

$$\gamma_i = \frac{g_i}{b_i + g_i} = \frac{g_i}{s_i}, \qquad 1 - \gamma_i = \frac{b_i}{b_i + g_i} = \frac{b_i}{s_i}. \tag{4}$$

Let G be the aggregate, or global, green investment:

$$G = \sum_{i=1}^{n} g_i. \tag{5}$$

We now come to describing the stochastic structure of returns on assets. For each type of assets $a \in \{b, g\}$, we use lower-case letters for logs: $r_a \equiv \ln R_a$. We make the following assumptions.

Assumption 1. *The asset returns satisfy the following decomposition:*

$$r_a = \widetilde{r}_a + z, \quad a \in \{b, g\}, \tag{6}$$

where z is the climate-related term (common for both assets), while $\widetilde{r}_a$ is the component determined by non-climate factors.

The climate finance literature does not appear to have converged to a consensus on the right terminology for the climate-related component z in the return equation (6). Some authors, including Giglio et al. (2021a,b), refer to it as a "climate shock" and assume that z can only take two values — zero and a strictly negative one — with endogenous probabilities. This is in line with the tradition of representing climate risks as low-probability events that have disastrous economic consequences (Barro, 2015; Wagner and Weitzman, 2016). An earlier version of our paper available here also follows that tradition. In the current version we relax the assumption that z is binomial and satisfies $z \leq 0$, but we still call z a climate shock.[1]

[1] Other authors use alternative terminology for z, for example, "innovation in climate news" (Engle et al., 2020). We prefer "climate shock" for the sake of brevity, although we acknowledge this is a slight abuse of language.

Assumption 2. *Changes in the cumulative green investment G shift the cdf $F(z \mid G)$ of the climate shock z, so that*

$$F(\cdot \mid G) \succ_{\text{FOSD}} F(\cdot \mid G') \Longleftrightarrow G > G',$$

where $\succ_{\text{FOSD}}$ is the first-order stochastic dominance ordering.

Assumption 3. *The joint distribution $(\widetilde{r}_b, \widetilde{r}_g)$ of non-climate return components is statistically independent of z and is not shifted by G.*

Assumption 4. *The function $\mathbb{E}[z|G]$ is concave w.r.t. G.*

Assumption 5. *The non-climate components $(\widetilde{r}_b, \widetilde{r}_g)$ of returns satisfy the following relation:*

$$\widetilde{r}_b = \widetilde{r}_g + \zeta,$$

where ζ is the greenium satisfying $\mathbb{E}[\zeta] > 0$.

Several comments are in order. First, Assumption 2 is equivalent to assuming that $F_G(z|G) < 0$ for all z, or that $\mathbb{E}_G[\varphi(z)|G] > 0$ for any increasing function $\varphi(\cdot)$. In particular, Assumption 2 implies that $\mathbb{E}[z|G]$ increases w.r.t. G.

Second, Assumption 3 essentially states that the distribution of the non-climate components of returns is unaffected by cumulative green investment G.

Third, Assumption 4 combined with Assumption 2 implies that $F(z \mid G)$ is decreasing and convex w.r.t. G for each z, meaning that there is diminishing marginal impact of global green investment on the climate change.

Finally, the existence of the greenium (Assumption 5) is supported by some empirical evidence. See, e.g., Grzegorczyk and Wolff (2022), Zerbib (2019) and Fatica et al. (2021). Taken together, the literature suggests that the greenium is real but modest in magnitude.[2]

We treat the joint distribution of returns $(R_b, R_g \mid G)$ as an exogenous object. A natural extensions would be to endogenize the returns by introducing demand and supply for various types of investment. However, we choose not to do that in this paper because our main results (see Proposition 3 below) remain true under endogenous returns. This is because we essentially do a cross-sectional comparison between agents in equilibrium, not a general-equilibrium-flavored comparative statics exercise.[3]

We now come to describing the preferences of agents.

Assumption 6. *Each agent i maximizes their expected lifetime utility function:*

$$\mathcal{U}_i(c_{i0}, c_{i1}, G) = u(c_{i0}) + \beta_i \mathbb{E}[u(c_{i1})|G], \tag{7}$$

where $u(\cdot)$ is the instantaneous utility function which is common across agents and satisfies the standard requirements of monotonicity ($u'(\cdot) > 0$) and concavity ($u''(\cdot) < 0$), $u'(\cdot) > 0$ while $\beta_i \in (0,1)$ is the discount factor, which can differ across agents.

[2] However, Larcker and Watts (2020) challenge the existence of a greenium in the US municipal bond market using a tight matching design, arguing that the green label per se commands no price premium when issuers and bond characteristics are properly controlled for.

[3] The only prediction of our model which would not necessarily survive in an environment with endogenous returns is the link between heterogeneity across agents and the amount of global green investment. We leave that for future research.

Assumption 6 achieves two purposes. First, it brings in the key ingredient of our approach — the investor-specific discount factor β_i which captures agent i's preference for the future. Second, it places the green investment decisions into the framework of an intertemporal portfolio choice problem. The next subsection highlights that our model has a hybrid nature and combines the features of a portfolio choice problem with those of a public good game.

2.2 Green investment as a public good game

One can view the global green investment G as a public good. This is because an increase in G has the effect of additional insurance against future climate events, which affects the future well-being of all agents. Thus, the problem each investor is facing has a hybrid nature: it blends features of an intertemporal portfolio choice problem with those of private provision of public goods in the spirit of Bergstrom et al. (1986). The next proposition formalizes that idea by providing a representation of how agents make interdependent green investment decisions as an outcome of a public good game.

Proposition 1. *Under Assumptions 1 — 6, decisions made by investors about savings s_i and green holdings g_i form a Nash equilibrium of a public good game given by*

$$\max_{(s_i,g_i)} \left\{ u\left(w_i - s_i\right) + \beta_i \mathbb{E}\left[u\left(R_b\left(s_i - \zeta g_i\right)\right) |G \right] \right\} \tag{8}$$

subject to:

$$G = g_i + \sum_{j \neq i} g_j; \tag{9}$$

$$0 \leq g_i \leq s_i \leq w_i. \tag{10}$$

Proof. See Appendix A.1. □

In Proposition 1, the first term in equation (8) captures the disutility of a reduction in present consumption due to more savings. The second term in (8) represents the tradeoff between private costs and public benefits of green investment: on the one hand, increasing the share of green holdings reduces future consumption due to the greenium ζ; on the other hand, a higher amount of global green investment leads to an increase (in the FOSD sense) of returns on both types of investments by mitigating the negative impact of the climate shock. The decisions of different agents are tied together through the constraint (9), which simply states that the global green investment can be decomposed into investor i's contribution and the contributions of others.

We now come to characterizing investment behavior.

Proposition 2. *Under Assumptions 1 — 6, one can represent agent i's FOCs as follows:*

- *the Euler condition:*

$$u'\left(c_{i0}\right) = \beta_i \mathbb{E}\left[R_b u'\left(c_{i1}\right) |G \right]; \tag{11}$$

- *the green investment condition:*

$$\frac{\partial \mathbb{E}\left[u\left(c_{i1}\right) |G \right]}{\partial G} = \mathbb{E}\left[\left(R_b - R_g\right) u'\left(c_{i1}\right) |G \right]. \tag{12}$$

Proof. See Appendix A.2. □

The Euler condition (11) is standard and captures the tradeoff between the present consumption and conventional investment. The green investment condition (12) states that the marginal gains in utility from allocating an additional euro to green investment need to be equal to the marginal cost of green investment. The latter manifests itself in the form of the covariance term between the marginal utility loss from less future consumption and the return differential $R_b - R_g$ capturing the greenium.

2.3 Key predictions: patience and wealth drive green investment

This subsection presents the predictions of the model about the impact of three key factors on green investment participation: investor's patience, their individual wealth, and wealth inequality across investors. To make things tractable, we introduce the following assumption in addition to Assumptions 2 — 6.

Assumption 7. *The instantaneous utility is logarithmic:* $u(c) = \log c$.

The following result reveals a link between the preference for the future, investor's wealth/size, and the share of green investment.

Proposition 3. *Under Assumptions 1 — 7, the Nash equilibrium in the green investment public good game* (8) *is unique and has the following properties:*

(i) *investor* i *is*

$$\text{a contributor } (g_i > 0) \Longleftrightarrow \frac{w_i \beta_i}{1+\beta_i} > \frac{\mathbb{E}\left[(R_b - R_g)/R_b\right]}{\partial \mathbb{E}\left[z|G\right]/\partial G}; \tag{13}$$

$$\text{a free-rider } (g_i = 0) \Longleftrightarrow \frac{w_i \beta_i}{1+\beta_i} \leq \frac{\mathbb{E}\left[(R_b - R_g)/R_b\right]}{\partial \mathbb{E}\left[z|G\right]/\partial G}; \tag{14}$$

(ii) *for any two investors,* i *and* j*,*

$$(w_i, \beta_i) \geq (w_j, \beta_j) \Longrightarrow \gamma_i \geq \gamma_j, \tag{15}$$

where $\gamma_{i,j}$ *are shares of green holdings in* i*'s and* j*'s portfolio structures defined by* (4)*;*

(iii) *eq.* (15) *holds with strict inequalities iff agent* i *is a contributor.*

Proof. See Appendix A.3. □

Proposition 3(i) provides a criterion for an agent to be a contributor or a free rider in the green investment public good game. The key indicator, $\frac{w_i \beta_i}{1+\beta_i}$, is increasing both w.r.t. agent's patience and their wealth. The threshold in the RHS of the separation rule (13) — (14) represents the marginal rate of substitution between the greenium and the insurance effect of green investment against the consequences of climate shocks. An agent is a contributor iff the indicator $\frac{w_i \beta_i}{1+\beta_i}$ exceeds that threshold. Hence, contributors are those agents who are either patient and not too small (in terms of wealth), or those who are wealthy and not too impatient. We can rewrite (13) — (14) in terms of elasticities and show that the ratio of $\frac{w_i \beta_i}{G(1+\beta_i)}$ needs to be larger than the elasticity of substitution between climate gains and the losses from greenium for an agent to be a contributor.

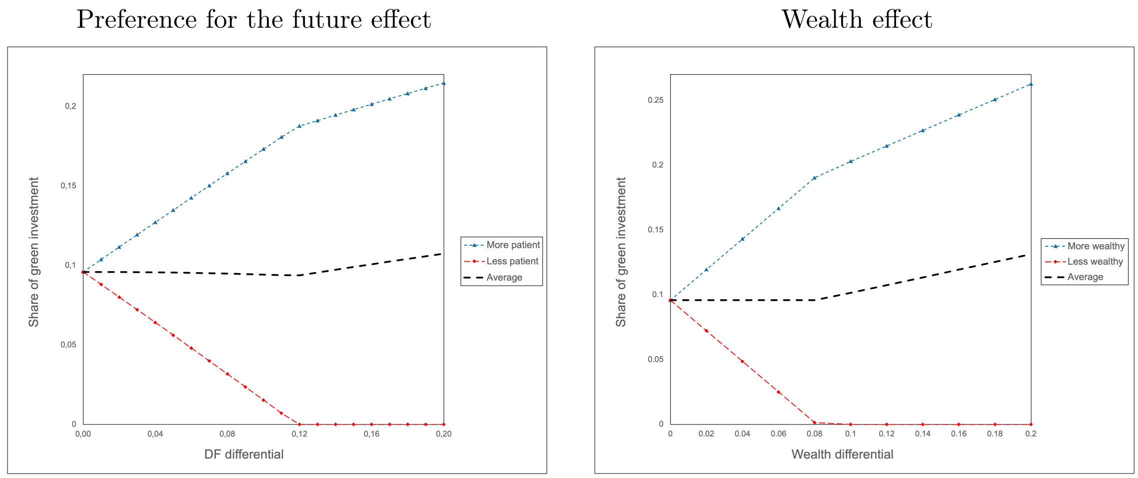


Figure 1: The role of preference for the future and wealth for green investment

Proposition 3(ii)-(iii) essentially states that agents with a higher preference for the future and higher wealth allocate higher shares of their savings into green assets.[4] Importantly, it is the *joint* impact of patience and wealth which matters: neither high preference for the future alone nor high wealth alone is enough for an investor to become a major contributor. Indeed, a long-term-oriented but small investor might choose not to contribute to the public good just because they do not expect to be able to change anything. Likewise, an extremely wealthy country whose authorities have a short planning horizon might also abstain from participation in climate change prevention. The US under its current leadership might provide an illustration for the latter scenario.

Figure 1 provides a numerical illustration of the results of Proposition 3 for a simple case of two investors and two possible states of the world: "a major climate event" and "the status quo". We simulate a simple mean-preserving increase in the difference between the two agents. The left panel of Figure 1 shows how individual green investments change in response to changing time preference. One can see that, first, the more long-term-oriented agent (that is, the one with a higher preference for the future) always holds a larger share of green assets than the less-patient agent, and that the share increases with the discount factor. The right panel of Figure 1 shows how individual green investments change in response to wealth. One can see that the wealthier agents invest more into green and second, that there is a threshold of wealth below which green investments become zero.

Figure 1 also illustrates a third important effect: the effect of an increase in wealth inequality. When the smaller agent's wealth becomes very low, their individual green investments fall to zero. This drives a third effect: the effect of large and increasing heterogeneity.[5] The next proposition provides more conditions for the model to predict that effect.

Proposition 4. *Under Assumptions 1 — 7, and under not too much heterogeneity across investors in preference for the future, a mean preserving spread of wealth leads to a higher equilibrium global green investment level.*

[4] In the LHS of (15), "$\leq$" denotes the partial order over $\mathbb{R}^2$. That is, the inequality holds component-wise.

[5] This result echoes the findings of (Bergstrom et al., 1986).

Proof. See Appendix A.4. □

2.4 Long-term investors are greener: a prediction in terms of observables

One can view the results of Proposition 3 as a theoretical rationalization of the empirical regularities discussed in the Introduction and in Section 3 below. However, it is fair to say that these results are not directly testable because discount factors are preference parameters, hence, they are not empirically observed. In our empirical analysis, we use the holdings of long-term assets in the investor's portfolio as an indicator of long-term oriented behavior. The theoretical counterpart of this indicator in our model is savings (as opposed to present consumption, which one can think of as short-term investment with negligibly small rates of return). This subsection discusses a testable prediction of our model — a positive relation between savings and green holdings.

Proposition 5. *Under Assumptions 1 — 7,*

(i) *for any two agents i and j,*

$$s_i \geq s_j \Longrightarrow \gamma_i \geq \gamma_j; \tag{16}$$

(ii) *if at least one of the two agents is a contributor, the following stronger version of* (16) *holds:*

$$s_i \gtreqless s_j \Longleftrightarrow \gamma_i \gtreqless \gamma_j; \tag{17}$$

(iii) *the relation between agent i's equilibrium savings rate and their green holdings is as follows:*

$$\frac{s_i}{w_i} = \frac{\beta_i\,(1+\mu_i)}{1+\beta_i\,(1+\mu_i)} \underbrace{\geq \frac{\beta_i}{1+\beta_i}}_{">" \Longleftrightarrow \gamma_i > 0}, \tag{18}$$

where μ_i is an agent-specific magnifying multiplier which is positively related to the share of green holdings and is positive/zero if agent i is a contributor/free rider.

Proof. See Appendix A.5. □

Parts (i)-(ii) of Proposition 5 demonstrate a positive relation between agent i's willingness of postpone consumption and save long-term, which is an empirically observable characteristic of long-term-oriented behavior, and the fraction of green holdings. This resonates positive correlation between investment in long-term assets and green holdings, which we find in Section 3.

Part (iii) of Proposition 5 illustrates the mechanism which links agent i's savings rate with their share of green holdings. From equation (18), one can see that this link operates through a magnification effect on the discount factor. That is, contributors to the public good are agents whose savings rates are higher than $\beta_i/(1+\beta_i)$, which is what the standard consumption-savings model with logarithmic preferences would predict.

2.5 Extension: CRRA preferences

The purpose of this section is to demonstrate the robustness of our main results stated in Proposition 3. To do this, we consider an extension of our baseline model with a richer parametric family of investors' preferences, namely, the constant relative risk aversion (CRRA) preferences, of which logarithmic utility is a special case.

Consider an extension of our baseline model in which the investors' preferences are characterized by constant relative risk aversion (CRRA). That is, we now relax Assumption 7 by replacing it with a more general one.

Assumption 8. *The instantaneous utility function $u(c)$ takes the following form:*

$$u(c) = \begin{cases} \frac{c^{1-\rho}-1}{1-\rho}, & \rho \neq 1, \\ \ln(c), & \rho = 1, \end{cases} \tag{19}$$

where $\rho > 0$ is the constant relative risk aversion.

From (19), the logarithmic utility is a special case of CRRA with $\rho = 1$.

One can show that agent i's future consumption increases/decreases/remains unchanged in response to more green investment made by others if and only if the relative risk aversion parameter ρ is greater than/smaller than/equal to one. To understand this finding, recall the well-known result from finance: high/low risk tolerance leads to more/less investment into a risky asset in response to a first-order stochastic dominance (FOSD) shock in the distribution of the returns on the risky asset. An increase in the aggregate green investment G clearly generates a FOSD shock in the probability distribution across the two possible future states of the world by causing a reduction in the probability of the extreme climate event.

The following proposition highlights the robustness of our main result to a departure from the logarithmic preference specification within the CRRA fammily of preferences.

Proposition 6. *Under Assumptions 1 — 6 and 8, agents with higher discount factors and/or higher wealth invest more and have higher shares of green holdings.*

Proof. See Appendix A.6. □

Thus, Proposition 6 shows that under CRRA preferences agents with higher preference for the future and/or greater wealth still invest more in green.

3 Stylized facts consistent with model predictions

In this section, we provide evidence and stylized facts on the relation between patience, economic size and green investments. First, we take advantage of a unique ECB database to provide evidence on the positive link between long-term orientation and green orientation of investors. Second, we explore the link between country size and climate mitigation efforts using the IRENA dataset for renewable energy investments.

3.1 Stylized fact 1: Patience and green investments

To get insights on whether patience matters for the holding of sustainable debt securities, we use the Securities Holdings Statistics (SHS) dataset by the European Central Bank (ECB). This dataset is collected on a security-by-security basis with quarterly data starting in 2021Q1, given in millions of current euros, and provides a breakdown by sector and sustainability classification (Green, Social, Sustainability and Sustainability-linked) as well as information on original maturity of the debt securities. The breakdown of sustainability holdings is available at the country level, while the breakdown by detailled holder sector is available for euro area aggregates. Based on the latter disaggregation, we consider whether, in fact, we observe an increase in holdings of sustainable securities in the euro area and which sectors are driving it.

The data are provided at different levels of aggregation. The first-digit level provides the economy as a whole broken down into non-financial corporations, financial corporations, general government, households, and non-profit institutions serving households. We then show the two digit breakdown of the financial sector, consisting of: 1. central banks, 2. deposit-taking corporations, except the central banks, 3. money market funds, 4. Non-MMF investment funds, 5. Financial vehicle corporations engaged in securitisation, 6. Insurance corporations, and 7. Pension funds.

Euro area holdings of sustainable debt as a share of total security holdings have increased markedly in recent years, in line with rapidly growing primary issuance in the sustainable-debt market (see Jena and Vuppuluri (2025), Dulak and Wolff (2026) and Figure 3 in the Appendix). The increase has taken place across the entire economy and all institutional sectors. In the first quarter of 2021, sustainable debt represented 5.04% of total holdings; by the second quarter of 2025 this share had risen by 8.8 percentage points to 13.9%, an average pace of roughly 2 percentage points per year. The increase is far from uniform across sectors: pension funds recorded the largest absolute gain (+17.3 pp over the period), followed by insurance corporations (+13.0 pp) and non-MMF investment funds (+12.2 pp), while money market funds (+1.1 pp) and general government (+5.7 pp) recorded more modest increases.

These patterns suggest that more patient, liability-driven investors, i.e., those whose mandate requires matching long-dated liabilities, could also be the most active participants in the sustainable-debt market.

To evaluate whether holders of sustainable debt also tend to hold long-maturity assets (original maturity $>$ 5 years), we perform a simple descriptive comparison for the most recent period: the share of originally long-term maturity assets against the share of sustainable assets. The exercise is performed for the whole economy and for the sub-sectors of the financial corporations' sector, as it is the sector that holds the most debt securities (91.2% of total economy holdings).

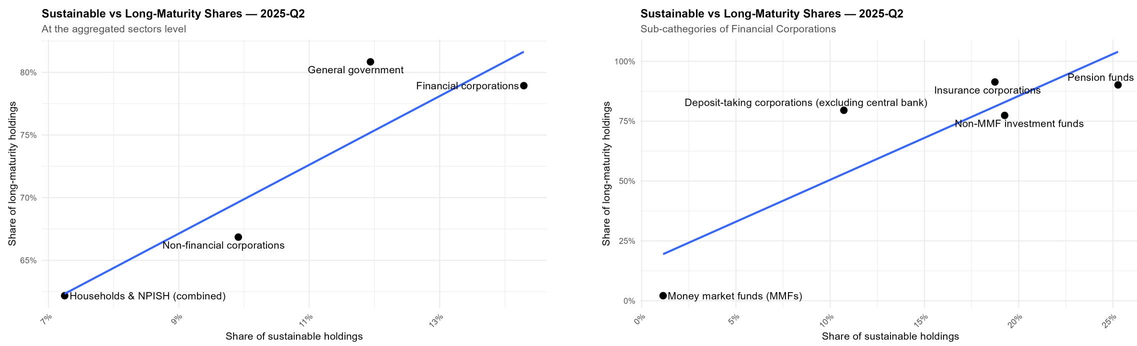
At the first-digit level, we see a positive relationship between the share of sustainable holdings and the share of long-term securities, see Figure 2a. At the two-digit level, different types of financial corporations[6] also display a slightly positive relationship, see Figure 2b.

To assess whether there is a stastically positive association, we estimate a two-way fixed-effects panel regression at the sector level over the eighteen quarters from 2021-Q1 to 2025-Q2, thereby exploiting the full dataset:

$$ShareGreen_{s,t} = \alpha_s + \gamma_t + \beta\, ShareLongTerm_{s,t-k} + \varepsilon_{s,t}, \tag{20}$$

where $ShareGreen_{s,t}$ is the share of sustainable debt in total security holdings of sector s in quarter t, and $ShareLongTerm_{s,t-k}$ is the corresponding share of securities with original maturity greater than five years, lagged $k \in \{0, 1, 2\}$ quarters. The sector fixed effects α_s absorb all time-invariant heterogeneity across sectors—investment mandate, risk appetite, and liability structure—while the quarter fixed effects γ_t absorb all sector-invariant aggregate shocks, including the secular expansion

[6] We exclude financial vehicle corporations engaged in securitisation, which only represent 2.2% of total financial assets, and the Central Bank.



(a) Share of sustainable assets in total asset holdings relative to the share of long-term investments (one-digit level of the economy). Data source: ECB Securities Holdings Statistics.

(b) Share of sustainable assets in total asset holdings relative to share of long-term assets, financial sector (two-digit level). Data source: ECB Securities Holdings Statistics.

Figure 2: Share of sustainable assets relative to long-term asset holdings across sectors.

of the sustainable-debt market and common regulatory changes affecting all sectors simultaneously. The coefficient β is therefore identified solely from *within-sector, within-quarter* variation: it captures whether a sector whose long-term holding share is above its own historical average in a given quarter simultaneously holds a disproportionately large share of sustainable assets, and vice versa. Standard errors are clustered at the sector level throughout.

In the contemporaneous specification ($k = 0$), the regressor and the dependent variable are measured in the same quarter. Because sustainable bonds are themselves predominantly long-dated instruments (typical original maturity of 7–12 years), a sector that purchases sustainable debt in quarter t mechanically raises both $ShareGreen_{s,t}$ and $ShareLongTerm_{s,t}$ within the same period, which could inflate the contemporaneous estimate. To address this, we also report specifications with $k = 1$ and $k = 2$: these use the sector's long-term holding share from one or two quarters prior to predict its current sustainable share, so that any contemporaneous sustainable-bond purchases cannot simultaneously affect the regressor.

Table 1 presents the results. Columns (1) and (2) use the contemporaneous long-term share on all fifteen ECB SHS reporting sectors and on thirteen sectors respectively, the latter excluding the two aggregate categories—total economy (S1) and total financial corporations (S12)—that are arithmetic sums of their sub-components and would otherwise introduce double counting. Columns (3) and (4) replicate the thirteen-sector specification with one- and two-quarter lags.

Across all four specifications, $\hat{\beta}$ is positive and statistically significant. In the baseline thirteen-sector specification (Column 2), a one percentage-point increase in a sector's long-term holding share is associated with a 0.16 percentage-point higher sustainable holding share. To understand the economic significance: moving from the twenty-fifth to the seventy-fifth percentile of the cross-sector long-term holding distribution—a shift of approximately 30 percentage points, corresponding roughly to the gap between non-MMF investment funds and money market funds—is associated with a $0.16 \times 30 \approx 5$ pp higher sustainable share. This gap amounts to roughly half the cross-sector standard deviation in sustainable holding shares observed in the data.

Table 1: Long-term investment and sustainable holdings: two-way fixed-effects panel estimates.

	Dependent variable: $ShareGreen_{s,t}$			
	(1)	(2)	(3)	(4)
	All sectors	13 sectors	13 sectors	13 sectors
	$k = 0$	$k = 0$	$k = 1$	$k = 2$
$ShareLongTerm_{s,t-k}$	0.164^{**}	0.162^{*}	0.145^{*}	0.124^{*}
	(0.080)	(0.084)	(0.080)	(0.075)
Sector FE	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes
R^2 (within)	0.120	0.117	0.106	0.090
Observations	270	234	221	208
Sectors (N)	15	13	13	13
Quarters (T)	18	18	17	16

Notes: Estimation period 2021-Q1 to 2025-Q2. All variables expressed as fractions (not percentages). Sector fixed effects absorb all time-invariant cross-sector heterogeneity; quarter fixed effects absorb all sector-invariant aggregate shocks, including the global expansion of the sustainable-debt market. Standard errors clustered at the sector level in parentheses. Column (2) excludes S1 (total economy) and S12 (total financial corporations) to avoid double counting, as these are aggregates of the remaining sectors. Columns (3)–(4) use *ShareLongTerm* lagged one and two quarters to address the mechanical overlap arising because sustainable bonds are themselves predominantly long-dated; both lagged specifications retain statistical significance, indicating the positive association is not solely mechanical. $^{***}p < 0.01$; $^{**}p < 0.05$; $^{*}p < 0.10$. We used Claude's co-work employing Python to run the regressions. Results were double-checked by replicating them in Microsoft's excel.

The magnitude of $\hat{\beta}$ declines slightly when using lags, but it remains significant at the 10% level throughout. This is consistent with a small mechanical component in the contemporaneous estimate arising from simultaneous purchases of long-dated sustainable bonds, along with a genuine structural relationship, which we predict in our model.

3.2 Stylized fact 2: economic scale and public renewable investment

When it comes to the effect of economic size on climate protection measures, the basic intuition is simple. Larger jurisdictions should internalise a larger part of their climate mitigation efforts and should therefore, ceteris paribus, be more willing to invest in climate protection. Schmidt et al. (2022) have aimed to test this hypothesis with carbon price data and population size. They cannot find evidence for a positive relation.

We carry out a similar exercise but focus on GDP as well as population size. Moreover, instead of carbon pricing, we look at actual investments in clean energy. Table 2 reports country and year fixed-effects regressions explaining renewable energy investments by country size and income. In particular, we run country and year fixed effect regressions of the log of public renewable energy investment on measures of economic scale, using IRENA data[7] for 149 countries over 2000–2023. Thanks to the fixed effects, we control for potentially country-specific differences in patience that our model predicts would be another factor determining green investments.

Specification (1) enters log(GDP total) alone; the coefficient is positive but statistically insignificant, because aggregate GDP conflates population scale and per-capita income whose effects partially offset. Specifications (2) and (3) decompose total GDP into its two components. The population coefficient rises to 2.68–2.76 and is highly significant ($p < 0.01$), while the per-capita income coefficient is small and insignificant ($p > 0.69$).

This decomposition reveals a clean *scale effect*: conditional on a country's income level, a larger population is associated with substantially more public renewable investment. Governance quality (WGI composite) is likewise insignificant within countries over time, consistent with its slow-moving nature being absorbed by country fixed effects. The addition of the governance quality control also does not change the results for the main variables. Importantly, we also control for time-fixed effects, which further reinforces the robustness of our results and ensures that the results are not driven by, for example, the rapidly falling prices of renewable energy.

The magnitude is economically large and, crucially, *overproportional*: a coefficient of $\hat{\beta} \approx 2.7$ implies that a country twice the population invests roughly $2^{2.7} \approx 6.5$ times as much in public renewables, far exceeding the proportional benchmark of $\beta = 1$. We formally reject proportional scaling ($H_0 : \beta_{\text{pop}} = 1$) in specification (2) at the 5% level ($t = 2.09$, $p = 0.037$) and at the 10% level in specification (3) ($t = 1.94$, $p = 0.052$). This pattern is consistent with the model's prediction that public green investment scales with jurisdiction size, as the public-goods component generates increasing returns to scale at the national level depending on size.

[7] IRENA (2024). Renewable Energy Finance Flows. International Renewable Energy Agency, Abu Dhabi. Available at: https://www.irena.org/Data/Downloads/IRENASTAT

Table 2: Economic Scale and Public Renewable Investment

	(1)	(2)	(3)
	Dependent variable: log(Public Renewable Investment)$_{c,t}$		
log(GDP total)$_{c,t}$	0.387		
	(0.542)		
log(Population)$_{c,t}$		2.681***	2.760***
		(0.805)	(0.907)
log(GDP per capita)$_{c,t}$		−0.097	−0.263
		(0.607)	(0.672)
WGI governance index$_{c,t}$			0.014
			(0.023)
Country FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Within R^2	0.000	0.007	0.007
Observations	2,278	2,278	2,215
Countries	149	149	148
$H_0: \beta_{\text{pop}} = 1$: t-stat		2.09	1.94
p-value (two-sided)		0.037	0.052

Notes: OLS with country and year fixed effects (within estimator). Sample: 149 countries including EU27 aggregate (EUU), 2000–2023; investment data from IRENA public renewable finance flows, 2022 USD million. Standard errors clustered by country with Cameron & Miller (2015) small-sample correction, reported in parentheses. Specification (3) loses one country due to missing WGI data. The row $H_0: \beta_{\text{pop}} = 1$ tests proportional scaling against the alternative that larger countries invest overproportionally more. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$. We used Claude's co-work employing Python to run the regressions. Results were double-checked by replicating them in Microsoft's excel.

4 Conclusions

We combine a public goods game with heterogenous agents differing in time preference and wealth/income with a portfolio choice model. Agents decide on investments in green or conventional bonds or on consumption. Although returns on green bonds are lower than on conventional bonds, they fund a public good G, which reduces the probability of an extreme climate event. The model provides three central empirical predictions. First, patience is a factor contributing to green investment. Second, larger economies invest more than smaller economies in green. Finally, a greater variance in the size of economies leads to larger green investments. With a simple empirical exercise, we can find support for the first two predictions.

Our results have important policy implications. First, the role of patience and long-term investment matters in financing the green transition. While policy makers cannot directly influence how patient investors are, they can facilitate long-term investments. Various regulatory requirements incentivize the holding of government debt while long-term assets with greater market uncertainty are relatively disadvantaged. A review and reform of regulatory burdens that render long-term investment less attractive could increase the share of portfolios that hold long-term debt and equity, thereby supporting green investments. A review of regulatory burdens on long-term investors that want to invest in green assets is also warranted.

Second, the size of a jurisdiction matters when it comes to its willingness to contribute to global climate mitigation. Building coalitions in the form of climate clubs should increase the probability that such larger coalitions pursue climate policies as the size of the deciding jurisdiction increases and thereby the extent to which the positive effects of climate policy are internalised. Moreover, the creation of clubs would also increase the heterogeneity of jurisdictions's size, possibly further increasing climate investments. Vosooghi et al. (2026) provide insights into how climate coalitions can be formed. The European Union perhaps exemplifies best such a strong club to foster climate mitigation. Overall, our paper makes the case that green finance and investments can be strengthened through long-term orientation and coalition building.

A final remark is in order. Due to its simple and parsimonious setup, our model can be used as a building block for richer settings tailored to analyzing specific aspects of climate investment. One possible line of future work could be considering a richer array of choices for green projects: agents can make adaptation investment and/or mitigation investment. Adaptation investments are purely private because they only protect capital of the agent who makes the investment, without affecting global environmental risks. On the contrary, mitigation investment has the nature of a public good because it reduces the probability of the extreme climate event. We leave this for future research.[8]

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[8] We have obtained some preliminary results on that front in the earlier version of our paper, in which there are only two states of the world and returns have no stochastic structure.

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Appendix

A Proofs

A.1 Proof of Proposition 1

(i) From (1) — (3), agent i's consumption path (c_{i0}, c_{i1}) satisfies

$$c_{i0} = w_i - s_i; \qquad c_{i1} = R_b s_i - (R_b - R_g) g_i. \tag{21}$$

From (6) and Assumption 5,

$$r_b - r_g = \ln\left(\frac{R_b}{R_g}\right) = \zeta \quad \Longrightarrow \quad \frac{R_b - R_g}{R_b} = 1 - e^{-\zeta} \approx \zeta, \tag{22}$$

where "$\approx$" stands for the first-order Taylor approximation, which is applicable since we assume (in accordance with empirical evidence) that the greenium ζ is small. Using (4) and (21) — (22), one can further restate the expression for c_{i1} as follows:

$$c_{i1} = R_b s_i \left(1 - \frac{(R_b - R_g)\, g_i}{R_b s_i}\right) = R_b s_i \left(1 - \zeta \gamma_i\right). \tag{23}$$

Plugging (21) into the utility function (7) and replacing $\frac{R_b - R_g}{R_b}$ with ζ, we obtain (8). Equation (9) is simply a restatement of (5), while inequalities (10) are self-explanatory. This completes the proof. □

A.2 Proof of Proposition 2

One can readily verify that the Euler condition (11) is simply the FOCs of agent i's utility maximization problem (8) — (10) w.r.t. s_i. To derive condition (12), consider agent i's FOC w.r.t. g_i:

$$u'\left(c_{i0}\right) = \beta_i \mathbb{E}\left[R_g u'\left(c_{i1}\right)|G\right] + \beta_i \frac{\partial \mathbb{E}\left[u\left(c_{i1}\right)|G\right]}{\partial G}.$$

Subtracting this FOC from the Euler condition (11) delivers (12). This completes the proof. □

A.3 Proof of Proposition 3

We start with the following lemma.

Lemma 1. *Under Assumptions 1 — 7, each agent i's backward best reply[9] in the aggregative game* (8) — (10) *is a single-valued function $\widehat{g}(G, \beta_i, w_i)$, which is non-negative-valued, continuous, piecewise differentiable, decreasing in G, increasing in both β_i and w_i, and satisfies*

$$\widehat{g}(G, \beta_i, w_i) > 0 \Longleftrightarrow \frac{w_i \beta_i}{1 + \beta_i} > \frac{\mathbb{E}\left[(R_b - R_g)/R_b\right]}{\partial \mathbb{E}\left[z|G\right]/\partial G}.$$

[9] The concept of the backward best reply, which refers to the collection of players' best-response strategies in the aggregative game compatible with a given level of the aggregator (in our case, the global green investment G), goes back to the pioneering work by Selten in the 1970s (see Selten (2013) for the latest edition). See Acemoglu and Jensen (2013) for a more general definition.

Proof. Let us first see how the investor's FOCs (11) — (12) look like under $u(c) = \ln c$ (Assumption 7). The Euler condition (11) becomes

$$\frac{1}{c_{i0}} = \beta_i \mathbb{E}\left[\left.\frac{R_b}{c_{i1}}\right| G\right], \tag{24}$$

while the green investment condition (12) takes the form

$$\mathbb{E}\left[\left.\frac{R_b - R_g}{c_{i1}}\right| G\right] = \frac{\partial \mathbb{E}\left[\ln c_{i1} | G\right]}{\partial G}. \tag{25}$$

Using (21) — (23), and taking into account that the distribution of ζ is not shifted by G (which follows from Assumptions 3 and 5), one can further simplify conditions (24) — (25), respectively, as follows:

$$s_i = \frac{\beta_i \left(1 + \mathcal{M}(\gamma_i)\right)}{1 + \beta_i \left(1 + \mathcal{M}(\gamma_i)\right)} w_i; \tag{26}$$

$$s_i = \frac{\mathcal{M}(\gamma_i)/\gamma_i}{\partial \mathbb{E}\left[z | G\right] / \partial G}. \tag{27}$$

where $\mathcal{M}(\gamma_i)$ is a magnifying multiplier given by

$$\mathcal{M}(\gamma_i) := \mathbb{E}\left[\frac{\zeta \gamma_i}{1 - \zeta \gamma_i}\right]. \tag{28}$$

Plugging the expression for s_i given by (26) into (27), we obtain the following fixed-point condition for γ_i:

$$\gamma_i = \frac{1}{w_i \frac{\partial \mathbb{E}[z|G]}{\partial G}} \left(\mathcal{M}(\gamma_i) + \frac{1}{\beta_i} \frac{\mathcal{M}(\gamma_i)}{1 + \mathcal{M}(\gamma_i)}\right). \tag{29}$$

The RHS of (29) is a strictly increasing function of γ_i which equals zero at $\gamma_i = 0$. Furthermore, the RHS of (29) is strictly convex w.r.t. γ_i. To see this, it suffices to show that $\frac{\mathcal{M}(\gamma_i)}{1+\mathcal{M}(\gamma_i)}$ is convex w.r.t. γ_i. From differentiating $\frac{\mathcal{M}(\gamma_i)}{1+\mathcal{M}(\gamma_i)}$ twice w.r.t. γ_i,

$$\frac{d^2}{d\gamma_i^2}\left[\frac{\mathcal{M}(\gamma_i)}{1 + \mathcal{M}(\gamma_i)}\right] = \frac{2}{\left(1 + \mathcal{M}(\gamma_i)\right)^3}\left[\frac{1}{2}\mathcal{M}''(\gamma_i)\left(1 + \mathcal{M}(\gamma_i)\right) - \mathcal{M}'^2(\gamma_i)\right]. \tag{30}$$

Let us now show that

$$\frac{1}{2}\mathcal{M}''(\gamma_i)\left(1 + \mathcal{M}(\gamma_i)\right) - \mathcal{M}'^2(\gamma_i) > 0. \tag{31}$$

Using (28), one can equivalently rewrite (31) as

$$\mathbb{E}\left[X_i^2\right]\mathbb{E}\left[Y_i^2\right] - \left(\mathbb{E}\left[X_i Y_i\right]\right)^2 > 0, \tag{32}$$

where

$$X_i := \frac{\zeta}{\left(1 - \zeta\gamma_i\right)^{3/2}}, \quad Y_i := \frac{1}{\left(1 - \zeta\gamma_i\right)^{1/2}}.$$

Inequality (32) is always true since it is the Cauchy-Schwarz inequality for random variables. Hence, (31) holds, and, from (30), the RHS of (29) is strictly convex w.r.t. γ_i. Hence, (29) has a unique interior solution iff the slope of the RHS of (29) at $\gamma_i = 0$ is strictly less than one. Otherwise, the only solution is the corner solution, $\gamma_i = 0$. One can easily verify that the slope of the RHS of (29) evaluated at $\gamma_i = 0$ is given by

$$\frac{1}{\frac{w_i \partial \mathbb{E}[z|G]}{\partial G}} \frac{d}{d\gamma_i} \left(\mathcal{M}(\gamma_i) + \frac{1}{\beta_i} \frac{\mathcal{M}(\gamma_i)}{1 + \mathcal{M}(\gamma_i)} \right) \bigg|_{\gamma_i = 0} = \frac{\mathbb{E}[\zeta]}{\frac{\partial \mathbb{E}[z|G]}{\partial G}} \frac{1 + \beta_i}{w_i \beta_i}.$$

Observe that conditions (13) — (14) are precisely the conditions for the slope of the RHS of (29) evaluated at $\gamma_i = 0$ to be less/greater than one. Thus, agent i is a contributor (resp., a free rider) iff condition (13) (resp., (14)) holds. Define $\widehat{\gamma}(G, \beta_i, w_i)$ and $\widehat{s}(G, \beta_i, w_i)$ as follows:

$$\widehat{\gamma}(G, \beta_i, w_i) := \begin{cases} \text{interior solution to (29)}, & \text{if (13) holds;} \\ 0, & \text{if (14) holds;} \end{cases} \tag{33}$$

$$\widehat{s}(G, \beta_i, w_i) = \frac{\beta_i \left(1 + \mathcal{M}(\widehat{\gamma}(G, \beta_i, w_i))\right)}{1 + \beta_i \left(1 + \mathcal{M}(\widehat{\gamma}(G, \beta_i, w_i))\right)} w_i; \tag{34}$$

$$\widehat{g}(G, \beta_i, w_i) := \widehat{\gamma}(G, \beta_i, w_i) \widehat{s}(G, \beta_i, w_i), \tag{35}$$

where $\widehat{g}(G, \beta_i, w_i)$ is agent i's backward best reply in the public good game (8) — (10). From Assumption 4, an increase in G results in an upward shift of the RHS of (29), while an increase in either β_i or w_i Since the the RHS of (29) is strictly convex w.r.t. γ_i, it follows that $\widehat{\gamma}(G, \beta_i, w_i)$ strictly decreases w.r.t. G and strictly increases w.r.t. both β_i and w_i whenever $\widehat{\gamma}(G, \beta_i, w_i) > 0$. This, combined with (34) and $\mathcal{M}'(\cdot) > 0$, implies that $\widehat{s}(G, \beta_i, w_i)$ also strictly decreases w.r.t. G and strictly increases w.r.t. both β_i and w_i. From (35), the same is true about $\widehat{g}(G, \beta_i, w_i)\widehat{s}(G, \beta_i, w_i)$ as long as $\widehat{s}(G, \beta_i, w_i) > 0$. Finally, from (33), $\widehat{g}(G, \beta_i, w_i) > 0$ holds iff (13) holds. This proves Lemma 1. □

We now proceed with the proof of Proposition 3.

To establish uniqueness of the Nash equilibrium in the public good game (8) — (10), plug $g_i = \widehat{g}(G, \beta_i, w_i)$ into the RHS of (5), which delivers a fixed-point condition for global green investment G:

$$G = \sum_{i=1}^{n} \widehat{g}(G, \beta_i, w_i). \tag{36}$$

From Lemma 1, the RHS of (36) is continuous and decreasing w.r.t. G. Hence, (36) has a unique solution $G^* > 0$ w.r.t. G. Agent i's Nash-equilibrium green investment level is then uniquely defined by $g_i^* := \widehat{g}(G^*, \beta_i, w_i)$.

It remains to notice that part (*i*) of Proposition parts (*ii*) — (*iii*) of Proposition 3 all follow immediately Lemma 1. This completes the proof. □

A.4 Proof of Proposition 4

We will only develop the proof for the case when there is no heterogeneity in time preference across investors, that is, when $\beta_1 = \beta_2 = \ldots = \beta_n = \beta$. By continuity, the result will be preserved in the case when there is not too much heterogeneity.

Let $\widehat{g}(G, \beta, w_i)$ be the restriction of agent i's backward best reply to the diagonal of the time preference domain ($\beta_i = \beta$). The following lemma constitutes the key ingredient of the proof.

Lemma 2. *Agent i's backward best reply* $\widehat{g}(G, w_i, \beta)$ *is convex w.r.t.* w_i.

Proof. We know from the proof of Proposition 3 that $\widehat{g}(G, w_i, \beta)$ is a continuous function of w_i, such that $\widehat{g}(G, w_i, \beta) = 0$ when w_i is below some threshold, and $\widehat{g}(G, w_i, \beta) > 0$ otherwise. Hence, it suffices to prove that $\widehat{g}(G, w_i, \beta)$ is convex w.r.t. w_i when it is strictly positive, that is,

$$\frac{\partial^2 \widehat{g}(G, \beta, w_i)}{\partial w_i^2} \geq 0. \tag{37}$$

The strictly positive part of the function $g_i = \widehat{g}(G, \beta, w_i)$ can be described by the following parametric equations, which follow immediately from (26) — (27):

$$w_i = \frac{1 + \beta \left[1 + \mathcal{M}(\gamma_i)\right]}{\beta \left[1 + \mathcal{M}(\gamma_i)\right]} \frac{\mathcal{M}(\gamma_i)/\gamma_i}{\partial \mathbb{E}\left[z|G\right]/\partial G}, \tag{38}$$

$$g_i = \frac{\mathcal{M}(\gamma_i)}{\partial \mathbb{E}\left[z|G\right]/\partial G}, \tag{39}$$

where γ_i serves as the parameterizing variable, while $\mathcal{M}(\gamma_i)$ is defined by (28). One can show by doing parametric differentiation that (37) holds. This proves Lemma 2. □

We now proceed with the proof of Proposition mean-preserving spread in the wealth distribution leads to an increase in the mean $\frac{1}{n}\sum_{i=1}^{n} \widehat{g}(G, w_i, \beta)$, hence to an upward shift of the RHS of (36). Since the RHS of (36) is decreasing in G, the unique solution G^* moves upwards along the 45-degree line, that is, the global green investment increases. This completes the proof of Proposition 4. □

A.5 Proof of Proposition 5

To prove parts (i) and (ii), observe first that, if i and j are both free riders, then (16) holds trivially, since $\gamma_i = \gamma_j = 0$. If i and j are both contributors, then, from (27) — (28), we get

$$s_k = \frac{\mathbb{E}\left[\zeta/(1 - \zeta\gamma_k)\right]}{\partial \mathbb{E}\left[z|G\right]/\partial G}, \quad k \in \{i, j\},$$

where the RHS is a strictly increasing function of γ_k, which implies (17). Finally, if one of the agents (say, i) is a contributor, while the other is a free-rider, then $\gamma_i > \gamma_j$, just because $\gamma_i > 0$ and $\gamma_j = 0$. Furthermore, from (26) and Proposition 3(i),

$$s_i > \frac{w_i\beta_i}{1 + \beta_i} > \frac{\mathbb{E}\left[(R_b - R_g)/R_b\right]}{\partial \mathbb{E}\left[z|G\right]/\partial G} \geq \frac{w_j\beta_j}{1 + \beta_j} = s_j.$$

Thus, $\gamma_i > \gamma_j$ and $s_i > s_j$ either hold simultaneously or neither of them holds, which implies that (17) holds. This proves parts (i) and (ii) of Proposition 5.

To prove part (iii), observe that relation (18) follows immediately from (26) (see the proof of Lemma 1 in Appendix A.3) if one sets $\mu_i := \mathcal{M}(\gamma_i)$. From (28), μ_i strictly increases w.r.t. γ_i and equals zero at $\gamma_i = 0$. This proves part (iii) and completes the proof of Proposition 5. □

A.6 Proof of Proposition 6

Under CRRA preferences (Assumption 8), the Euler condition (11) and the green investment condition (12) become, respectively, as follows:

$$c_{i0}^{-\rho} = \beta_i \mathbb{E}\left[R_b c_{i1}^{-\rho} \middle| G \right]; \tag{40}$$

$$\mathbb{E}\left[(R_b - R_g)\, c_{i1}^{-\rho} \middle| G \right] = \frac{1}{1-\rho} \frac{\partial \mathbb{E}\left[c_{i1}^{1-\rho} \middle| G \right]}{\partial G}. \tag{41}$$

Rewriting (40) — (41) in terms of investor i's savings s_i and investor i's green holdings γ_i, we obtain the following equations:

$$s_i = w_i \frac{\left(\beta_i \mathbb{E}\left[R_b^{1-\rho} | G \right] \right)^{1/\rho} \mathcal{M}(\gamma_i, G)}{\left(\beta_i \mathbb{E}\left[R_b^{1-\rho} | G \right] \right)^{1/\rho} \mathcal{M}(\gamma_i, G) + 1}, \tag{42}$$

$$s_i = \frac{1-\rho}{\partial \mathbb{E}\left[e^{(1-\rho)z} | G \right] / \partial G} \frac{\mathbb{E}\left[\zeta R_b^{1-\rho} (1 - \zeta\gamma_i)^{-\rho} \middle| G \right]}{\mathbb{E}\left[e^{(1-\rho)\tilde{r}_b} (1 - \zeta\gamma_i)^{1-\rho} \right]}, \tag{43}$$

where

$$\mathcal{M}(\gamma_i, G) := \left(\frac{\mathbb{E}\left[R_b^{1-\rho} \left(\frac{1}{1-\zeta\gamma_i} \right)^{\rho} \middle| G \right]}{\mathbb{E}\left[R_b^{1-\rho} \middle| G \right]} \right)^{1/\rho}.$$

The RHS of (43) is clearly increasing w.r.t. γ_i. Hence, investors with larger savings also have large green holdings. Furthermore, the RHS of (42) clearly increases w.r.t. β_i and w_i, which means investors with higher discount factors and/or higher wealth levels invest more. This completes the proof. □

B Figures

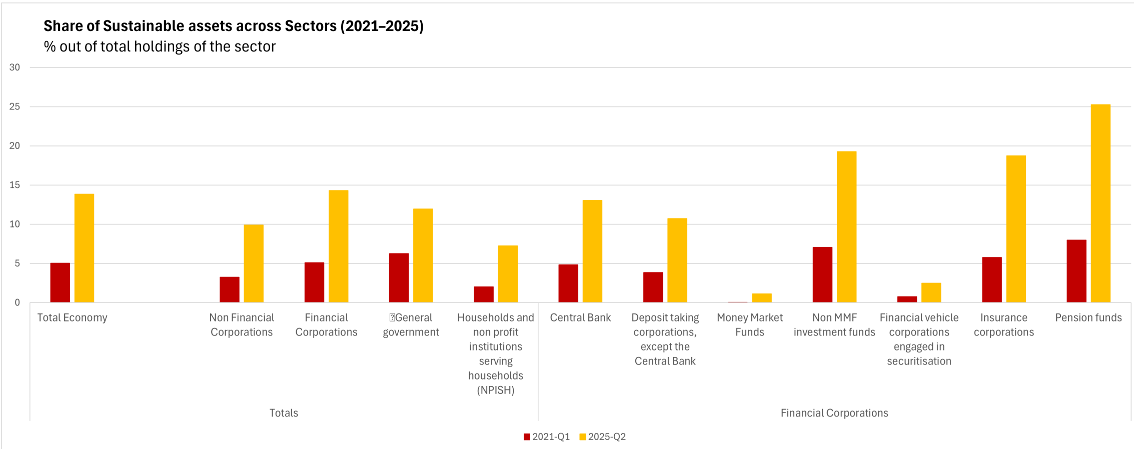

Figure 3: Share of sustainable assets in total asset holdings. Data source is ECB's Security Holdings statistics dataset